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Abstract

We consider the propagation and source distribution of cosmic rays. The principal requirement for the various models we consider is that they should be capable of holding particles in dense regions of the galactic disk for periods of time sufficient to produce the observed fragmentation products of cosmic rays. This can be achieved by both simple and compound diffusion provided that suitable mean free paths and boundary conditions are chosen. The bulk of the anisotropy is caused by the discrete nature of the cosmic ray sources. However, models which reproduce the fragmentation products will in general yield anisotropies consistent with available upper limits.

Invited paper presented by R. Ramaty at the Discussion on the Isotopic Composition of Cosmic Ray Nuclei, Lyngby, Denmark, March 23 to March 25, 1971.

INTRODUCTION

The concept of diffusive motion of cosmic rays was first proposed by Fermi (1949). Diffusion equations for cosmic ray acceleration and propagation were subsequently treated by several authors (e.g. Fan 1951, Morrison, Olbert and Rossi 1954). Syrovatskii (1959) wrote down and gave a general solution for a transfer equation which included spatial diffusion, sudden losses and energy exchange.

The problem of nuclear fragmentation in interstellar space in terms of such a transfer equation was treated in detail by Ginzburg and Syrovatskii (1964). Because of its mathematical complications, however, subsequent treatments of nuclear fragmentation have ignored this model and have replaced the details of galactic propagation by the simple statement that the fragmentation and energy loss of cosmic rays are the result of the passage of the particles through a slab of matter of given thickness x, measured in grams per cm^2. Studies of the charge composition of cosmic rays (e.g. Shapiro and Silberberg 1970), however, have demonstrated that this slab approximation is not adequate to account for the observed fragmentation products of both the CNO and the iron group nuclei and that the observed elemental abundances could be better understood in terms of a distribution P(x) of matter traversals x.

As a possible form for P(x), Cowsik et al. (1967) have suggested an exponential distribution for the potential path lengths of the particles from their sources to earth. This exponential distribution, however, is simply the result of the replacement of the diffusive term $D\Delta n$ (D is the diffusion coefficient and n is the cosmic ray density)

by an escape term n/τ. The resultant transfer equation (e.g. Morrison 1961) has been widely used for the study of the propagation of cosmic electrons in interstellar space (Hayakawa and Okuda 1962, Gould and Burbidge 1965, Ramaty and Lingenfelter 1966). Ramaty and Lingenfelter (1968) have also used this equation for the propagation of cosmic ray deuterons and helium-3 nuclei.

While the problem of nuclear fragmentation can be treated without explicit consideration of spatial dependences, the problem of cosmic ray anisotropy clearly depends on the spatial and temporal properties of the source distribution and particle propagation. It was first suggested by Baade and Zwicky (1934) that cosmic rays may predominantly be produced by supernova explosions. This point of view has been substantially strengthened (Shklovsky 1953, Ginzburg 1953) by the discovery of non-thermal radio emission from supernova remnants. More recently, the discovery of pulsars gave additional support to this point of view, and even though the detailed mechanism of cosmic ray production is as yet unknown, it is quite generally accepted that the production of the bulk of the cosmic rays at earth above a few tens of MeV/nucleon is associated with the explosive event of supernova formation.

The implications of a cosmic ray source distribution which is a superposition of discrete events in space-time has been considered by Lingenfelter (1969), Jones (1970a), and Ramaty, Reames and Lingenfelter (1970).

In Lingenfelter's (1969) treatment the distances and ages of supernovae were replaced by the estimated distances and ages of known pulsars. In the treatments of Jones (1970a) and of Ramaty, Reames, and Lingenfelter (1970) a random distribution of supernovae in space and time was used. The principal conclusion of these treatments was that the possible values of cosmic ray anisotropy undergo large statistical fluctuations as a result of the uncertainty in positions and ages of the cosmic ray sources. Jones (1970a,b) chose to interpret the high degree of cosmic ray isotropy in terms of such fluctuations while Ramaty, Reames and Lingenfelter (1970) preferred a model in which the low anisotropy is the result of the slow propagation of the cosmic rays. While in the treatment of Ramaty, Reames and Lingenfelter (1970) this slow propagation was achieved by simple 3-dimensional diffusion with a short mean free path, Lingenfelter, Ramaty and Fisk (1971) showed that the same result can be obtained by compound diffusion with a much larger mean free path.

In the present paper we shall treat in a consistent fashion the problems of fragmentation and anisotropy within specific models for cosmic ray propagation and source distribution. We limit our discussion to propagation by diffusion, both simple and compound. We shall formulate general transfer equations for cosmic rays which take into account these forms of propagation as well as nuclear fragmentation and energy loss. We consider both continuous and discrete source distributions and present the solutions of the transfer equations for ultrarelativistic as well as mildly relativistic particles. We consider in detail the

fragmentation of ultrarelativistic nuclei from iron to lithium and we present the age distributions which best fit the observed fragmentation products. Finally, for the same formalism and models, we calculate the anisotropy and compare our results with observed upper limits.

Particle Propagation and Transfer Equations for Cosmic Rays

There is considerable observational evidence, (Morris and Berge 1964, Hornby 1966, Davies 1967) that on a scale smaller than about 1 kpc the interstellar magnetic field is quite disordered, even though on a larger scale the field appears to lie nearly along the spiral arms. The major observed irregularities in the interstellar medium are interstellar gas clouds which have a mean diameter of around 10 pc, a mean separation of about 40 pc and a mean distance between clouds of roughly 100 pc along an arbitrary line of sight (Allen 1963). The interstellar magnetic field may therefore be considered as random with a scale size ℓ which could range from several parsecs to a few hundred parsecs. Since the gyroradii of the majority of cosmic rays are much smaller than ℓ, cosmic-ray particles will follow field lines for distances up to and comparable to ℓ, and may be transferred to other field lines thereafter. The resultant motion is random walk which can be approximated by diffusion provided that ℓ is smaller than the linear dimension of the confinement volume of the cosmic rays.

The validity of the diffusion approximation for the propagation of cosmic rays in this type of magnetic field distribution has been discussed in detail by Ginzburg and Syrovatskiǐ (1964). The transfer equation for the number density $n(\vec{r}, t, \varepsilon)$, which allows for spatial diffusion, energy loss and fragmentation of a particular nuclear component of the cosmic rays, is given by

$$\frac{\partial n}{\partial t} - \nabla\cdot(D\nabla n) + \frac{\partial}{\partial \varepsilon}(Bn) + \frac{n}{T} = Q(\vec{r},t,\varepsilon) \qquad (1)$$

Here $D = 1/3\, lc\beta$ is the diffusion coefficient, $\vec{r}, t, \varepsilon$ and $c\beta$ are position, time, kinetic energy per nucleon and velocity, respectively, $B = d\varepsilon/dt$ is the rate of energy loss per nucleon, and $T^{-1} = T_d^{-1} + T_e^{-1}$, where T_d is the nuclear-collision loss time and T_e is an escape time which in some cases will be used in lieu of specific boundary conditions. The source term $Q(\vec{r},t,\varepsilon)$ is in general position, time and energy dependent and consists of both primary sources, where nuclei are accelerated to cosmic ray energies, and secondary sources, where nuclei are produced by fragmentation. Equation (1) is valid for cosmic electrons as well, provided that $T_d \to \infty$, ε is total kinetic energy and $d\varepsilon/dt$ is the energy loss rate appropriate for electrons.

In the subsequent discussion, we shall assume that D, B, and T are independent of $\vec{r}$ and t and are functions of ε alone. The solution of equation (1) can then be written as (Syrovatskii 1959)

$$n(\vec{r},t,\varepsilon) = \int_{-\infty}^{t} dt_0 \frac{B(\varepsilon_0)}{B(\varepsilon)} \exp\left[-\int_{\varepsilon}^{\varepsilon_0}\frac{d\varepsilon'}{BT}\right] \int d^3r_0\, Q(\vec{r}_0,t_0,\varepsilon_0) f(\vec{r},\vec{r}_0,\tau)\,. \qquad (2)$$

Here, the integral d^3r_0 is over the spatial extent of the source distribution Q; f $(\vec{r}, \vec{r}_0, \tau)$ satisfies the equation

$$\frac{\partial f}{\partial \tau} - \nabla^2 f = \delta(\vec{r}-\vec{r}_0)\,\delta(\tau) \qquad (3)$$

with the same boundary conditions as $n(\vec{r}, t, \varepsilon)$; and ε_0 and τ are determined by

$$t + t_0 = \int_{\varepsilon}^{\varepsilon_0} \frac{d\varepsilon'}{B(\varepsilon')} \quad ; \quad \tau = \frac{1}{D_0}\int_{\varepsilon}^{\varepsilon_0} \frac{D(\varepsilon')\,d\varepsilon'}{B(\varepsilon')} \qquad (4)$$

where D_0 is the diffusion coefficient at some fixed energy and it is assumed that the particles always lose energy so that $B > 0$ and $\varepsilon_0 > \varepsilon$.

The solutions of equation (3) depend on the assumed boundary conditions. For cosmic ray propagation in the galaxy, it is convenient to use a coordinate system with the x and y axes in the plane of the disk and the z axis perpendicular to this plane. If we assume that the diffusing medium is infinite in the x and y direction and has absorbing or reflecting boundaries at $z = \pm\, a$, the solution of equation (3) becomes

$$f_{\mp}(\vec{r}, \vec{r}_0, \tau) = \left(\frac{3}{4\pi \ell_{\parallel} c\tau}\right)\left(\frac{3}{4\pi \ell_{\perp} c\tau}\right)^{\frac{1}{2}} \exp\left[-\frac{3(x-x_0)^2 + 3(y-y_0)^2}{4\ell_{\parallel} c\tau}\right] \times$$
$$\times \left\{\sum_{n=-\infty}^{\infty} \exp\left[-\frac{3(|z-z_0| - 4na)^2}{4\ell_{\perp} c\tau}\right] \mp \sum_{n=-\infty}^{\infty} \exp\left[-\frac{3(|z+z_0| + 4na - 2a)^2}{4\ell_{\perp} c\tau}\right]\right\} \qquad (5)$$

where (-) or (+) correspond to absorbing or reflecting boundaries, respectively, $\vec{r} = (x, y, z)$, $\vec{r}_0 = (x_0, y_0, z_0)$, and $\ell_{\parallel}$ and $\ell_{\perp}$ are the

mean free paths parallel and perpendicular to the galactic plane, respectively. If $a \to \infty$ and $\ell_{\parallel} = \ell_{\perp}$ equation (5) reduces to isotropic diffusion in an infinite medium,

$$f(\vec{r}, \vec{r}_0, \tau) = \left(\frac{3}{4\pi \ell c \tau}\right)^{3/2} \exp\left[-\frac{3|\vec{r}-\vec{r}_0|^2}{4\ell c \tau}\right] \tag{6}$$

Before presenting the evaluation of equation (2) with the diffusion solutions (5) and (6), we shall consider a possible departure from simple 3-dimensional diffusion. In the above discussion we pointed out that 3-dimensional diffusion is the result of motion in a disordered magnetic field with scale size ℓ in which particles follow field lines for distances $s \lesssim \ell$, and are transferred in a random manner to other field lines for $s > \ell$. Such irregularities could be produced by the cosmic rays themselves, or by some additional source of turbulence in the interstellar medium. These irregularities could scatter the cosmic rays, but as long as the gyroradii of the particles are smaller than λ, this scattering is essentially one dimensional along the local field lines. The resultant motion, which combines one-dimensional diffusion along field lines with three-dimensional random walk, is compound diffusion (Lingenfelter, Ramaty, and Fisk 1971). It should be pointed out that compound diffusion could result even if there are no small scale irregularities superimposed on the random field. In this case, the additional scattering along the field lines would be the consequence of the bends of scale size ℓ in the random field itself.

When compound diffusion is taken into account, the transfer equation (1) has to be replaced by the pair of equations

$$\frac{\partial}{\partial t} n(\vec{r},t,\varepsilon,s) - \frac{1}{3}\lambda c\beta \frac{\partial^2 n}{\partial s^2} + \frac{\partial}{\partial \varepsilon}(Bn) + \frac{n}{T} = \int d^3 r_o\, Q(\vec{r}_o,t,\varepsilon) f_2(\vec{r},\vec{r}_o,s) \tag{7}$$

and

$$\frac{\partial f_2(\vec{r},\vec{r}_o,s)}{\partial s} - \frac{1}{3}\ell \nabla^2 f_2 = \delta(\vec{r}-\vec{r}_o)\,\delta(s) \tag{8}$$

where s is a linear distance measured along field lines. Equations (7) and (8) can be solved by the same methods as equation (1). If we chose the observation point to be at s = o, the solution can be written as

$$n(\vec{r},t,\varepsilon) = \int_{-\infty}^{t} dt_o \frac{B(\varepsilon_o)}{B(\varepsilon)} \exp\left[-\int_{\varepsilon}^{\varepsilon_o} \frac{d\varepsilon'}{BT}\right] \times$$

$$\times \int d^3 r_o\, Q(\vec{r}_o,t_o,\varepsilon_o) \int_0^{\infty} ds_o\, f_1(s_o,\tau) f_2(\vec{r},\vec{r}_o,s_o) \tag{9}$$

where f_1 satisfies the one-dimensional diffusion equation

$$\frac{\partial f_1}{\partial \tau} - \frac{1}{3}\lambda c \frac{\partial^2 f_1}{\partial s^2} = \delta(s)\,\delta(\tau) \tag{10}$$

and ε_o and τ are determined by equations (4). Equation (9) for compound diffusion is equivalent to equation (2) for simple three-dimensional diffusion provided that

$$f(\vec{r},\vec{r}_o,\tau) = \int_0^{\infty} ds_o\, f_1(s_o,\tau) f_2(\vec{r},\vec{r}_o,s_o) \tag{11}$$

If one-dimensional scattering along the field lines is negligible $(\lambda \to \infty)$, $f_1 \to \delta(s_0 - c\tau)$, so that $f_2(\vec{r}, \vec{r}_0, s_0) = f(\vec{r}, \vec{r}_0, \tau)$ and equations (2) and (7) become in fact identical.

The functions f_1 and f_2 depend on the boundary conditions used to solve equations (8) and (10). For simplicity we assume infinite diffusing media so that

$$f(\vec{r}, \vec{r}_0, \tau) = \int_0^\infty ds \left(\frac{3}{\pi \lambda c \tau}\right)^{\frac{1}{2}} \exp\left[-\frac{3s^2}{4\lambda c \tau}\right] \left(\frac{3}{4\pi \ell s}\right)^{\frac{3}{2}} \exp\left[-\frac{3|\vec{r}-\vec{r}_0|^2}{4\ell s}\right] \quad (12)$$

Lingenfelter, Ramaty, and Fisk (1971) have evaluated this integral as a function of τ/τ_0, where $\tau_0 = |\vec{r}-\vec{r}_0|^4/\ell^2\lambda c$ is the time to maximum for compound diffusion. The results are shown in Figure 1 together with the equivalent curve for simple 3-dimensional diffusion. As can be seen, the decay time for compound diffusion is much longer than for simple diffusion. Similarly, the rise time is also much larger, since the values of τ_0 differ by a factor of $2(\vec{r}-\vec{r}_0)^2/\ell\lambda$ which is much larger than 1 if the distance from source to observer greatly exceeds the mean free path. The net result of these differences is that cosmic ray propagation in interstellar space is much slower by compound diffusion than by simple diffusion with the same values of the mean free paths.

COSMIC RAY DENSITIES AND AGE DISTRIBUTIONS

Given the source function Q, the local interstellar density can be obtained by evaluating equation (2) or (9), for simple or compound diffusion, respectively. As discussed above, Q consists of both primary and secondary sources. The primary sources are believed to be discrete events in space and time such as supernova explosions or pulsars. For such discrete events, the source distribution can be written as

$$Q(\vec{r},t,\varepsilon) = \sum_m Q_m(\varepsilon)\,\delta(\vec{r}-\vec{r}_m)\,\delta(t-t_m) \tag{13}$$

where $\vec{r}_m$ and t_m are the position and time of occurence of a particular source and m runs over the assumed source distribution. By substituting equation (13) into equations (2) or (9) we get

$$n(\vec{r},t,\varepsilon) = \sum_m \frac{B(\varepsilon_m)}{B(\varepsilon)}\,Q_m(\varepsilon_m)\exp\left[-\int_{\varepsilon}^{\varepsilon_m}\frac{d\varepsilon'}{BT}\right] f(\vec{r},\vec{r}_m,\tau_m) \tag{14}$$

where ε_m and τ_m are given by equations (4) with ε_0 and τ replaced by ε_m and τ_m , and f is given by equations (5) or (11), for simple or compound diffusion, respectively.

Ramaty, Reames and Lingenfelter (1970) have evaluated equation (14) for a random source distribution. They have shown that whereas the cosmic-ray anisotropy undergoes large fluctuations as a result of the discrete and variable source distribution, the cosmic-ray density remains relatively constant. We shall consider these matters in some detail below, after the treatment of nuclear fragmentation.

The secondary source consists of particles produced by nuclear collisions of primary cosmic rays with interstellar gas. Because of the relative stability of the interstellar density of primary particles, it is possible to assume that the secondary source is time independent and a separable function of position and energy, i.e.

$$Q(\vec{r},t,\varepsilon) = q(\varepsilon)\,\rho(\vec{r}) \tag{15}$$

The energy dependent part $q(\varepsilon)$, depends on nuclear cross sections and kinematics as well as on the spectrum of the primary particles which produce the secondary nuclei in question. The position dependent part, ρ, which is chosen to be a non-dimensional function of $\vec{r}$, depends on the spatial distribution of both the interstellar gas the the equilibrium density of the primary cosmic rays. Since both these distributions are expected to peak in the galactic plane, we assume that

$$\rho(\vec{r}) = \begin{cases} 1 & \text{for } |x| < \infty,\ |y| < \infty,\ |z| < b \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

where b is approximately equal to the scale height of interstellar hydrogen. We use $b = 100$ pc. By substituting equation (16) into equations (2) or (9) we get

$$n(\vec{r},\varepsilon) = \int_0^\infty d\tau\, \frac{\beta(\varepsilon_0)B(\varepsilon_0)}{\beta(\varepsilon)B(\varepsilon)}\, q(\varepsilon_0) \exp\left[-\int_\varepsilon^{\varepsilon_0} \frac{d\varepsilon'}{BT}\right] \times \int d^3r_0\, f(\vec{r},\vec{r}_0,\tau)\,\rho(\vec{r}_0) \tag{17}$$

where ε_0 and τ are given by equations (4) and $c\beta(\varepsilon)$ is particle velocity at energy per nucleon ε.

We consider now the evaluation of equation (17). We first separate T into a destruction time T_d and an escape time T_e. T_d is energy dependent and is given by

$$T_d = \left[n_H c \beta \sigma(\varepsilon) \right]^{-1} \tag{18}$$

where σ is the destruction cross section and n_H is the interstellar hydrogen density. In the calculation of T_d we shall assume that n_H is a constant over the confinement volume of the cosmic rays. The escape time T_e may also be energy dependent. We assume, however, that the mean free paths ℓ and λ are energy independent, so that the only energy dependence of T_e is through β, i.e.

$$T_e = \tau_e \beta^{-1} \tag{19}$$

where τ_e is a constant. With these assumptions, equation (17) can be written as

$$n(\vec{r},\varepsilon) = \int_0^\infty d\tau \, \frac{d\varepsilon_0/dx}{d\varepsilon/dx} \, q(\varepsilon_0) \exp\left[-\int_\varepsilon^{\varepsilon_0} \frac{d\varepsilon' \sigma(\varepsilon') dx/d\varepsilon'}{m_p} \right] P(\tau) \tag{20}$$

where m_p is the proton mass, $d\varepsilon/dx = (m_p n_H c \beta)^{-1} B(\varepsilon)$ and

$$P(\tau) = \exp(-\tau/\tau_e) \, P_0(\tau) \tag{21}$$

where

$$P_0(\tau) = \int d^3 r_0 \, f(\vec{r}, \vec{r}_0, \tau) \, \rho(\vec{r}_0) \quad . \tag{22}$$

$P(\tau)$ is the age distribution of the cosmic rays. Using equation (16) for ρ, we can evaluate $P_o(\tau)$ from equation (22) for any propagation function f. For simple diffusion, with f given by equation (5), we get

$$P_o(\tau) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\left\{\mathrm{erf}\left[\frac{3(b+z-4na)^2}{4lc\tau}\right]^{1/2} + \mathrm{erf}\left[\frac{3(b-z-4na)^2}{4lc\tau}\right]^{1/2}\right\}$$
$$\mp\frac{1}{2}\sum_{n=-\infty}^{\infty}\left\{\mathrm{erf}\left[\frac{3(b+z+4na-2a)^2}{4lc\tau}\right]^{\frac{1}{2}} + \mathrm{erf}\left[\frac{3(b-z+4na-2a)^2}{4lc\tau}\right]^{1/2}\right\} \tag{23}$$
$$\pm\sum_{n=-\infty}^{\infty}\mathrm{erf}\left[\frac{3(4na-2a)^2}{4lc\tau}\right]^{1/2}$$

where the upper and lower signs correspond to absorbing and reflecting boundaries, respectively, and z is the height of the observation point above the galactic plane. If we use equation (6) instead of equation (5) for f (or if we let $a \to \infty$ in equation (23)), we get $P_o(\tau)$ for diffusion in an infinite medium

$$P_o(\tau) = \frac{1}{2}\left\{\mathrm{erf}\left[\frac{3(b+z)^2}{4lc\tau}\right]^{1/2} + \mathrm{erf}\left[\frac{3(b-z)^2}{4lc\tau}\right]^{1/2}\right\} . \tag{24}$$

For compound diffusion, with f givenby equation (12), we obtain

$$P_o(\tau) = \int_0^\infty ds\left(\frac{3}{\pi\lambda c\tau}\right)^{\frac{1}{2}}\exp\left(-\frac{3s^2}{4\lambda c\tau}\right)\frac{1}{2}\left\{\mathrm{erf}\left[\frac{3(b+z)^2}{4ls}\right] + \mathrm{erf}\left[\frac{3(b-z)^2}{4ls}\right]\right\} \tag{25}$$

As can be seen from equations (23), (24), and (25), $P_o(\tau)$, and hence also the age distribution $P(\tau)$, depend on the distance z of the point of observation from the galactic plane. In order to investigate this dependence, we have evaluated the quantity $\int_0^\infty P(\tau)\,d\tau$. If nuclear collisions and energy losses are neglected, this integral equals to the

ratio between the equilibrium density and production rate per unit volume, i.e., from equation (20)

$$n(\vec{r},\varepsilon) = q(\varepsilon)\int_0^\infty P(\tau)\,d\tau \tag{26}$$

Therefore, the quantity $\int_0^\infty P(\tau)\,d\tau$, which has temporal dimensions, may be defined as the trapping time of the cosmic rays in the confinement volume. The trapping time in general differs from the mean age of the particles given by $\int_0^\infty \tau P(\tau)\,d\tau \Big/ \int_0^\infty P(\tau)\,d\tau$, a fact which should be kept in mind when comparing the abundances of secondary fragmentation products with the survival probabilities of certain radioactive nuclei.

The spatial dependences of the trapping time and its gradient are plotted in Figure 2. The implications of the gradient on the anisotropy will be discussed below. Here we merely note that for the evaluation of the cosmic ray densities at earth, we can assume that z = 0, since the variation of $\int_0^\infty P(\tau)\,d\tau$ from z = 0 to the present position of the solar system of z = 10 pc is almost entirely negligible.

The distributions $P_0(\tau)$ for z = 0 are shown in Figure 3 as functions of τ/τ_0. As can be seen, for simple diffusion with absorbing boundaries P_0 decreases rapidly with τ/τ_0. Since the observed ratio of light to medium nuclei requires a trapping time of a few million years, for simple diffusion with absorbing boundaries, the mean free path ℓ cannot be larger than a few tenths of parsecs. This, however, is not the case for reflecting boundaries where P_0 approaches the constant value of b/a

and the trapping time goes to infinity unless the age distribution is cutoff at some large value of τ by a finite escape probability at the boundary. In fact, for values of ℓ much larger than a few tenths of parsecs, P_0 becomes independent of ℓ and depends only on the assumed escape probability. In the present paper, we did not introduce such an escape probability as an explicit boundary condition. Instead, we use the exponential term $\exp(-\tau/\tau_e)$ to take into account the effects of this escape.

Also shown in Figure 2 are the distributions P_0 for diffusion in an infinite medium and compound diffusion. The infinite medium case clearly lies in between the absorbing and reflecting boundary cases and will not be further considered in this paper. The parameters appropriate for compound diffusion will be discussed below.

COSMIC RAY FRAGMENTATION

We consider now the problem of cosmic ray fragmentation using the age distributions derived above. We limit our discussion to relativistic nuclei so that energy losses can be neglected and we assume that all cross sections are energy independent. Under these assumptions, equation (20) reduces to

$$n(\varepsilon) = q(\varepsilon)\int_0^{\infty} d\tau \exp\left[-n_H c\sigma\tau\right] P(\tau) \tag{27}$$

Let $j = 1, \ldots, N$ range over all stable isotopes with $j = 1$ corresponding to protons and $j = N$ to Fe^{56}. In the fragmentation of nuclei heavier than hydrogen or helium, it is reasonable to assume that the velocities, and hence also the energies per nucleon of the fragments, are the same as those of the incident nucleus. Therefore, the equilibrium density n_j of isotope j resulting from the fragmentation of isotopes i with equilibrium densities n_i $(i > j)$ can be written as

$$n_j(\varepsilon) = q_j(\varepsilon)\,\tau_j \tag{28}$$

where

$$q_j(\varepsilon) = \sum_{i=j+1}^{N} n_H c\sigma_{ij}\, n_i(\varepsilon)$$

$$\tau_j = \int_0^\infty d\tau \exp\left[-n_H c\sigma_{jj}\tau\right] P(\tau) \tag{29}$$

and σ_{ij} and σ_{jj} are the fragmentation cross sections of isotope i into isotope j and the total breakup of isotope j, respectively.

Since measurements of individual isotopes are not yet available, the computational technique must depend on the comparison of the calculated and measured fluxes of individual elements. Let $j_1 \leq j \leq j_2$ range over all isotopes with the same atomic number Z. The equilibrium density n_Z of element Z can then be written as

$$n_Z = \sum_{j=j_1}^{j_2} n_j' + \sum_{j=j_1}^{j_2}\sum_{i=j+1}^{j_2} c n_H \sigma_{ij} n_i' \tau_j + \ldots +$$

$$+ N_k \left\{ 1 + (cn_H)\sum_{j=j_1}^{k-1} \sigma_{kj}\tau_j + (cn_H)^2 \sum_{j=j_1}^{k-2}\sum_{i=j+1}^{k-1} \sigma_{ki}\tau_i\sigma_{ij}\tau_j + \ldots \right\} \tag{30}$$

where k ($j_1 \leq k \leq j_2$) is the isotope of element Z which is most likely produced by nucleosynthesis in the primary sources and

$$n_j' = \sum_{i=j_2+1}^{N} n_H c \sigma_{ij} n_i \tau_j \tag{31}$$

The first term on the right hand side of equation (30) is the sum of the partial densities n_j' produced by the fragmentation of isotopes with atomic numbers greater than Z, while the second term results from the tertiary interactions of these partial densities. The third, fourth and fifth terms on the r.h.s. represent, respectively, the primary, secondary, and tertiary densities of the isotope k. Equation (30) can be solved for N_k in terms of the measured density n_Z. Using this value of N_k, we can express the densities of the individual isotopes of element Z as follows:

$$n_j = n_j' + \sum_{i=j+1}^{j_2} c n_H \sigma_{ij} n_i' \tau_j + N_k \left\{ 1 + c n_H \sigma_{kj} \tau_j + (c n_H)^2 \sum_{i=j+1}^{k-1} \sigma_{ki} \tau_i \sigma_{ij} \tau_j \right\} \tag{32}$$

where the second term vanishes if $j = j_2$, the third term is zero for all j except j = k, and the fourth and fifth terms vanish if $j > k-1$ and $j > k-2$, respectively.

The cross sections σ_{ij} were compiled by D. V. Reames and will be published elsewhere (Ramaty, Reames and Lingenfelter, in preparation).

Using these cross sections, we have evaluated equation (32) with the partial densities n_i' given by equation (31), N_k obtained from equation (30), the observed densities n_Z as summarized by Shapiro and Silberberg (1970), and the quantities τ_j given by equation (29) for a variety of age distributions $P(\tau)$. The results, summed over individual elements, are given in Table 1, together with the observed abundances n_Z. Models (1) and (2) are 3-dimensional diffusion with absorbing and reflecting boundaries, respectively. Model (3) is compound diffusion and model (4) represents an exponential age distribution, i.e.

$$P(\tau) = \exp(-\tau/\tau_e) \tag{33}$$

The parameters for the various models are summarized in Table 1. These parameters were chosen so that the value of Li, Be and B to the measured mean densities of C, N and O equals 0.23 (Shapiro and Silberberg 1970).

The age distributions $P(\tau)$ for the same parameters are plotted in Figure 4. As can be seen the distributions based on diffusion and disk geometry predict more particles at large τ than the exponential distribution. This is the result of the geometry we use in which the thickness of the trapping volume is larger than that of the source, so that particles produced close to the plane can reenter the central parts of the disk after a relatively long trapping time in the confinement volume.

Using the primary equilibrium abundances N_k as determined from equation (30), we can deduce the source abundances q_k,

$$q_k = N_k / \tau_k \tag{34}$$

The results for the various models are given in Table 2. As can be seen the source abundances of some elements are negative, indicating that the fragmentation process produces too many secondaries for that element. By considering the uncertainties in the observations, however, none of these negative abundances are significant. Nevertheless, an age distribution with an upturn at large values of τ will tend to produce relatively less fragmentation products of the iron group for the same L/M ratio than the exponential distribution. This effect is most pronounced for the distributions corresponding to simple diffusion with reflecting boundaries and compound diffusion.

COSMIC RAY ANISOTROPY

There are a number of reasons to expect anisotropy in the cosmic ray intensity. Compton and Getting (1935) first pointed out that, even if the cosmic ray intensity were isotropic in some rest frame other than that of the earth, the intensity measured in the moving frame of the earth would be anisotropic. In a detailed derivation Gleeson and Axford (1968) show that in the relativistic limit this anisotropy in the moving frame may be written

$$\delta = (2+\Gamma)\, v/c \tag{35}$$

where the differential intensity is assumed to be a power law in kinetic energy per nucleon with exponent Γ, and v is the velocity of the moving frame.

Thus if the cosmic rays were isotropic in the frame of the interstellar medium and its associated magnetic field, the motion of the solar system relative to this frame will cause an anisotropy. With respect to the neighboring stars the sun has a velocity of about 20 ± 0.5 km sec^{-1} in the direction of right assension $271° \pm 2°$ and declination $+ 30 \pm 1°$, or galactic longitude $\ell = 57 \pm 1°$ and latitude $b = + 22 \pm 2°$ (Allen 1963). In the energy range from about 10^{11} to 10^{15} eV where $\Gamma = 2.65$ we see from equation (35) that the anisotropy from this effect would be about 3×10^{-4}. This is equal to the upper limit set by Elliot et al. (1970) at 10^{11} to 10^{12} eV. Since the gyroradius of 10^{11} to 10^{12} eV particles in the interplanetary magnetic field is on the order of 1 A.U., the low anisotropy observed at these energies is probably not representative of its interstellar value. Nevertheless, the cosmic ray anisotropy in the interstellar medium should not exceed the value of 10^{-3} determined at about 2×10^{13} eV (Cachon 1963) where the gyroradius in the interplanetary field is about 100 A.U. so that the effects of this field can be neglected.

An anisotropy is also expected from the propagation of the cosmic rays. The bulk of this anisotropy is due to the discrete nature of the cosmic ray source distribution. However, even if this distribution were continuous and time independent, an anisotropy would result from the asymmetric position of the solar system with respect to the source

distribution and the boundaries.

We can estimate the magnitude of this anisotropy from Figure 1. For a = 300 pc, we see that $(1/n)(\partial n/\partial z) \simeq 3\times10^{-4}\,\mathrm{pc}^{-1}$ at $z = 10\,\mathrm{pc}$. The anisotropy due to this density gradient is given by $\ell/n\ \partial n/\partial z$. Since Figure 1 was derived for 3-dimensional diffusion with absorbing boundaries, we use $\ell \simeq .1\,\mathrm{pc}$. The resultant anisotropy of $\sim 3 \times 10^{-5}$ is much smaller than the minimum anisotropy which could be expected from the Compton-Getting effect associated with the motion of the sun.

In order to investigate the anisotropy resulting from the discrete-source nature of the cosmic ray sources, we have to use the source distribution given by equation (13). For relativistic nuclei energy losses can be neglected and all destruction cross sections become energy independent. Equation (14) can then be written as

$$n(\vec{r},t) = \sum_m Q_m \exp\left(-\frac{t-t_m}{T}\right) f(\vec{r},\vec{r}_m,t-t_m) \tag{36}$$

For ordinary diffusion the anisotropy is given by

$$\delta = \frac{3D}{c}\frac{|\nabla n|}{n} = \ell\frac{|\nabla n|}{n} \tag{37}$$

By substituting equation (36) with f given by equation (5) into equation (37) we get

$$\begin{aligned}\delta = \frac{1}{n}\Bigg\{&\left[\sum_m \frac{3}{2}\frac{x-x_m}{c(t-t_m)} Q_m \exp\left(-\frac{t-t_m}{T}\right) f(\vec{r},\vec{r}_m,t-t_m)\right]^2 + \\ &\left[\sum_m \frac{3}{2}\frac{y-y_m}{c(t-t_m)} Q_m \exp\left(-\frac{t-t_m}{T}\right) f(\vec{r},\vec{r}_m,t-t_m)\right]^2 + \\ &\left[\sum_m \ell\frac{\partial}{\partial z} f(\vec{r},\vec{r}_m,t-t_m)\, Q_m \exp\left(-\frac{t-t_m}{T}\right)\right]^2\Bigg\}^{1/2}\end{aligned} \tag{38}$$

where $\partial f/\partial z$ is given by

$$\frac{\partial f}{\partial z} = \left[\frac{3}{4\pi \ell c(t-t_m)}\right]^{3/2} \exp\left[-\frac{3(x-x_m)^2+3(y-y_m)^2}{4\ell c(t-t_m)}\right] \times$$
$$\times\left\{\sum_{n=-\infty}^{\infty} \frac{3(4na-|z-z_m|)}{2\ell c(t-t_m)} \frac{\partial}{\partial z}|z-z_m| \exp\left[-\frac{3(|z-z_m|-4na)^2}{4\ell c(t-t_m)}\right]\right.$$
$$\left.\mp \sum_{n=-\infty}^{\infty} \frac{3(4na-2a+|z+z_m|)}{2\ell c(t-t_m)} \frac{\partial}{\partial z}|z+z_m| \exp\left[-\frac{3(|z+z_m|+4na-2a)^2}{4\ell c(t-t_m)}\right]\right\} \tag{39}$$

where (-) and (+) correspond to absorbing and reflecting boundaries, respectively.

We have evaluated equations (38) and (39) for a random distribution of cosmic ray sources in the galactic disk of semithickness 100 pc normalized to a galactic rate of 1 event per hundred years. The mean anisotropies as functions of ℓ are plotted in Figure 5 for absorbing and reflecting boundaries. The error bars represent $\pm 1\sigma$ levels, i.e., the probability that for a given value of ℓ the anisotropy will be bracketed by the error bar is 0.66. The anisotropies for the absorbing-boundary case are quite similar to the anisotropies calculated by Ramaty Reames, and Lingenfelter (1970) for diffusion in an infinite medium with exponential escape. As ℓ increases, a small number of young sources with large anisotropies contribute the bulk of the local cosmic-ray flux and hence the anisotropy increases rapidly with increasing ℓ. In the case of reflecting boundaries and large values of ℓ, the same young sources will still have large anisotropies, but now their contribution to the local flux is much smaller and hence the total anisotropy is much lower.

For the upper limit of 10^{-3} (Cachon 1962), the model with absorbing boundaries requires a value of ℓ smaller than about 1 pc. From the study of nuclear fragmentation, we found that for this model ℓ is about 0.06 pc, which would allow anisotropies as low as 10^{-4}. In the model with reflecting boundaries, for $\ell \gtrsim 1$ pc the amount of nuclear fragmentation becomes independent of ℓ. The upper bounds on the anisotropy would require, however, that ℓ be less than about one hundred parsecs.

Finally, we consider the anisotropy if cosmic rays propagate by compound diffusion. Since in this case, the effective diffusion coefficient is space and time dependent in a non-separable way, we can no longer use equation (37) to compute the anisotropy. The streaming $\vec{S}$, and the related anisotropy $\delta = 3|\vec{S}|/cn$, however, can be directly obtained from the continuity equation

$$\frac{\partial n}{\partial t} + \nabla \cdot \vec{S} + \frac{n}{T} = 0 \quad . \tag{40}$$

Let $\vec{S}$ be given in terms of its components from individual sources

$$\vec{S}(\vec{r}\ t) = \sum_m \exp\left(-\frac{t-t_m}{T}\right) \vec{J}(\vec{r}, \vec{r}_m, t-t_m) \tag{41}$$

By substituting equations (36) and (41) into equation (40) we get

$$\frac{\partial f(\vec{r}, \vec{r}_m, t-t_m)}{\partial t} + \nabla \cdot \vec{J}(\vec{r}, \vec{r}_m, t-t_m) = 0 \tag{42}$$

As above, we limit our discussion to isotropic compound diffusion in an infinite medium. Using equation (11) for f, the magnitute of $\vec{J}$ from equation (42) can be written as

$$|\vec{J}(\vec{r},\vec{r}_m,t-t_m)| = \frac{2}{\xi^2}\int_0^\infty ds\,\frac{\partial f_1(s,\tau)}{\partial\tau}\int_0^\xi \xi'^2 f_2(\xi',s)\,d\xi' \qquad (43)$$

where $\xi = |\vec{r}-\vec{r}_m|$ and $\tau = t-t_m$.

By using the forms of f_1 and $f_2$appropriate for infinite diffusive media, equation (43) becomes

$$|\vec{J}| = \frac{1}{2\pi\xi^2}\left(\frac{3}{4\pi\lambda c\tau}\right)^{\frac{1}{2}}\frac{1}{\tau}\int_0^\infty ds\left(\frac{3\xi^2}{4\lambda c\tau}-\frac{1}{2}\right)\exp\left[-\frac{3s^2}{4\lambda c\tau}\right] I_{1/2}\left[\left(\frac{3}{2}\right)^{\frac{1}{2}}\frac{\xi^2}{2\lambda s}\right] \qquad (44)$$

where

$$I_{1/2}(u) = 1/\Gamma\left(\tfrac{3}{2}\right)\int_0^{\sqrt{3/2}\,u} v^{1/2}e^{-v}\,dv \qquad (45)$$

(Pearson 1957).

When the numerical evaluation of equation (44) is combined with that of equation (12), for all values of $\tau > \tau_0$, the anisotropy from a single source, $\delta(\xi,\tau) = 3|\vec{J}|/cf$, is very closely given by

$$\delta = 3\xi/4c\tau \qquad (46)$$

This result should be compared with $\delta = 3\xi/2c\tau$ for simple diffusion.

Using equation (41) and (46), the anisotropy from a distribution of sources is given by

$$\delta = \frac{3}{4cn}\Bigg\{\left[\sum_m \frac{x-x_m}{t-t_m} Q_m \exp\left(-\frac{t-t_m}{T}\right) f(\vec{r},\vec{r}_m,t-t_m)\right]^2 +$$

$$\left[\sum_m \frac{y-y_m}{t-t_m} Q_m \exp\left(-\frac{t-t_m}{T}\right) f(\vec{r},\vec{r}_m,t-t_m)\right]^2 +$$

$$\left[\sum_m \frac{z-z_m}{t-t_m} Q_m \exp\left(-\frac{t-t_m}{T}\right) f(\vec{r},\vec{r}_m,t-t_m)\right]^2 \Bigg\}^{\frac{1}{2}} \qquad (47)$$

where n is given by equation (36) and f by equation (12).

We have evaluated equation (47) for the same random distribution of cosmic ray sources as used in conjunction with equation (38) for simple diffusion. For $\lambda = \ell = 30$ pc and $\tau_e = 2 \times 10^7$ years.

$$\delta = \left(0.7 \begin{smallmatrix} +2.0 \\ -0.5 \end{smallmatrix}\right) \times 10^{-4} \qquad (48)$$

where the fluctuations represent $\pm 1\sigma$ levels as discussed above. As can be seen, for compound diffusion, the anisotropy is quite small and almost negligible in comparison with the Compton-Getting effect resulting from the motion of the sun with respect to the local interstellar gas and field.

CONCLUSION

We have investigated the effects of the source distribution and propagation on the composition and anisotropy of cosmic rays. For the calculation of the composition we used a continuous and time independent

source distribution of semithickness 100 pc in the galactic disk. For the anisotropy we used a statistical discrete source model with cosmic ray sources occurring at random in a disk of similar spatial extent. As specific propagation modes, we considered simple 3-dimensional diffusion with boundary conditions and compound diffusion in an infinite medium with exponential escape. We found that in order to account for the observed abundances of the fragmentation products of nuclei from lithium to iron, the mean free path for simple diffusion with absorbing boundaries cannot be larger than about 0.1 pc. For diffusion with reflecting boundaries, the amount of fragmentation depends principally on the escape probability at the boundary and is almost independent of the mean free path. However, the upper limit on the anisotropy requires a mean free path in this model of less than about one hundred parsecs. For compound diffusion, both the fragmentation and anisotropy can be accounted for if the characteristic length in the 3-dimensional random field is about 30 pc, the mean free path for 1-dimensional diffusion along field lines is equal to, or smaller than this value, and the escape time from the confinement volume is about 2×10^7 years.

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CAPTION FOR TABLES

1. Charge spectrum of cosmic rays from Li to Fe. The observed abundances were taken from Shapiro and Silberberg (1970). The calculated abundances are the result of nuclear fragmentation with $n_H = 1\ cm^{-3}$.

   Model 1: 3-dimensional diffusion with absorbing boundaries at b = 300 pc, $\ell = 0.06$ pc and $\tau_e \to \infty$.

   Model 2: 3-dimensional diffusion with reflecting boundaries at b = 200 pc, $\ell = 30$ pc and $\tau_e = 1.4 \times 10^7$ years.

   Model 3: Compound diffusion with $\ell = \lambda = 30$ pc and $\tau_e = 2 \times 10^7$ years.

   Model 4: Exponential age distribution with $\tau_e = 3.3 \times 10^6$ years ($X = 5\ g/cm^2$).

2. Source abundances of cosmic rays from Li to Fe. The parameters for the various models were described in the caption to Table 1.

CAPTION FOR FIGURES

1. Time profiles for simple 3-dimensional and compound diffusion. The quantities τ_o are times to maximum.

2. The cosmic ray density and its gradient as a function of distance from the plane of the disk, z. The length a is the semithickness of the trapping volume.

3. Age distributions P_o defined in equation (22) for various models. The quantity τ_o are times to maximum for the distance b, the semi-thickness of the source distribution.

4. Age distributions P defined by equation (21) for various models. The appropriate parameters are given in the caption to Table 1.

5. The anisotropy δ as a function of the mean free path ℓ. The upper and lower curves are for simple diffusion with absorbing and reflecting boundaries, respectively.

<u>TABLE 1</u>

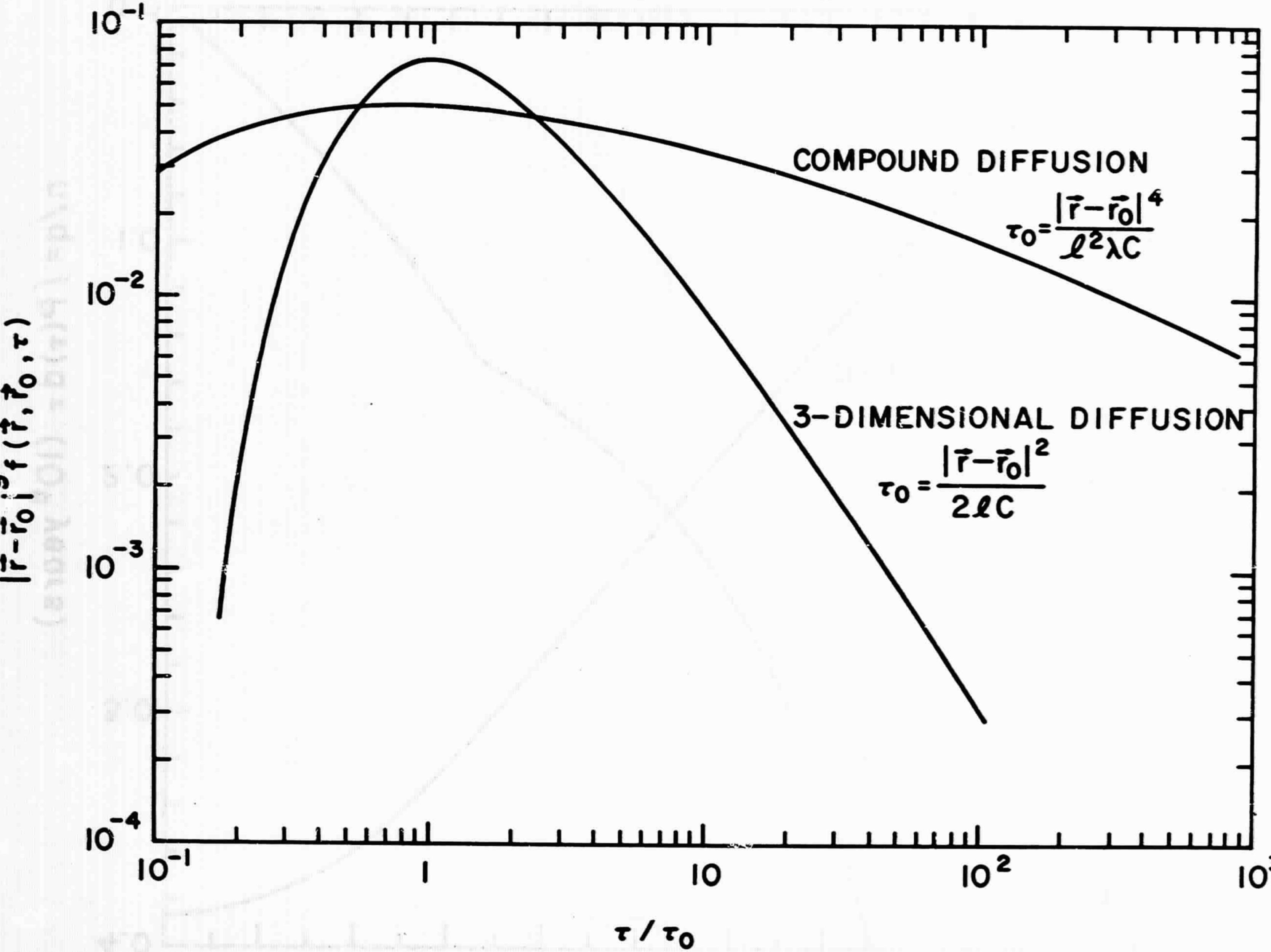
FRAGMENTATION PRODUCTS

Z	Element	Observed	Model 1	Model 2	Model 3	Model 4
26	Fe	1 ± .12	0	0	0	0
25	Mn	.08 ± .03	.030	.023	.027	.033
24	Cr	.31 ± .09	.096	.070	.086	.10
23	V	.09 ± .03	.088	.065	.079	.096
22	Ti	.18 ± .05	.17	.13	.15	.18
21	Sc	.027 ± .02	.059	.045	.053	.064
20	Ca	.18 ± .05	.17	.12	.15	.18
19	K	.053 ± .027	.12	.090	.11	.14
18	Ar	.18 ± .05	.13	.097	.11	.14
17	Cl	.044 ± .026	.062	.048	.056	.068
16	S	.31 ± .09	.14	.11	.13	.15
15	P	$.053 \pm {}^{.12}_{.044}$	.046	.037	.042	.050
14	Si	1.33 ± .18	.15	.12	.13	.16
13	Al	.18 ± .09	.099	.081	.091	.11
12	Mg	1.86 ± .18	.28	.23	.26	.30
11	Na	.27 ± .14	.20	1.66	.18	.21
10	Ne	1.8 ± .2	.46	.39	.43	.49
9	F	.18 ± .1	.26	.22	.24	.27
8	O	7.6 ± .35	.58	.51	.54	.61
7	N	2.4 ± .18	1.28	1.17	1.22	1.34
6	C	8.8	1.10	1.03	1.06	1.15
5	B	2.4 ± .26	2.04	1.97	2.00	2.10
4	Be	.97 ± .26	.69	.69	.68	.69
3	Li	1.42 ± .18	1.56	1.68	1.62	1.54

TABLE 2

SOURCE ABUNDANCES

Z	Element	Model 1	Model 2	Model 3	Model 4
26	Fe	1	1	1	1
25	Mn	.048	.055	.051	.045
24	Cr	.21	.23	.22	.20
23	V	.002	.023	.011	-.006
22	Ti	.007	.046	.023	-.007
21	Sc	-.029	.015	-.023	-.033
20	Ca	.010	.045	.025	-.003
19	K	-.059	-.031	-.047	-.070
18	Ar	.042	.063	.052	.032
17	Cl	-.014	-.0029	-.0095	-.019
16	S	.14	.15	.14	.13
15	P	.0052	.011	.0082	.0022
14	Si	.89	.83	.88	.89
13	Al	.056	.061	.060	.051
12	Mg	1.11	1.01	1.10	1.11
11	Na	.048	.060	.057	.040
10	Ne	.85	.77	.84	.85
9	F	-.048	-.024	-.037	-.059
8	O	4.14	3.44	3.96	4.27
7	N	.62	.55	.62	.62
6	C	4.03	3.14	3.76	4.23
5	B	.17	.16	.19	.15
4	Be	.13	.094	.12	.14
3	Li	-.056	-.068	-.068	-.053



COMPOUND DIFFUSION
$\tau_0 = \frac{|\vec{r}-\vec{r}_0|^4}{\ell^2 \lambda C}$
3-DIMENSIONAL DIFFUSION
$\tau_0 = \frac{|\vec{r}-\vec{r}_0|^2}{2\ell C}$
$|\vec{r}-\vec{r}_0|^3 f(\vec{r},\vec{r}_0,\tau)$
τ/τ_0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-1}
1
10
10^2
10^3

Figure 1

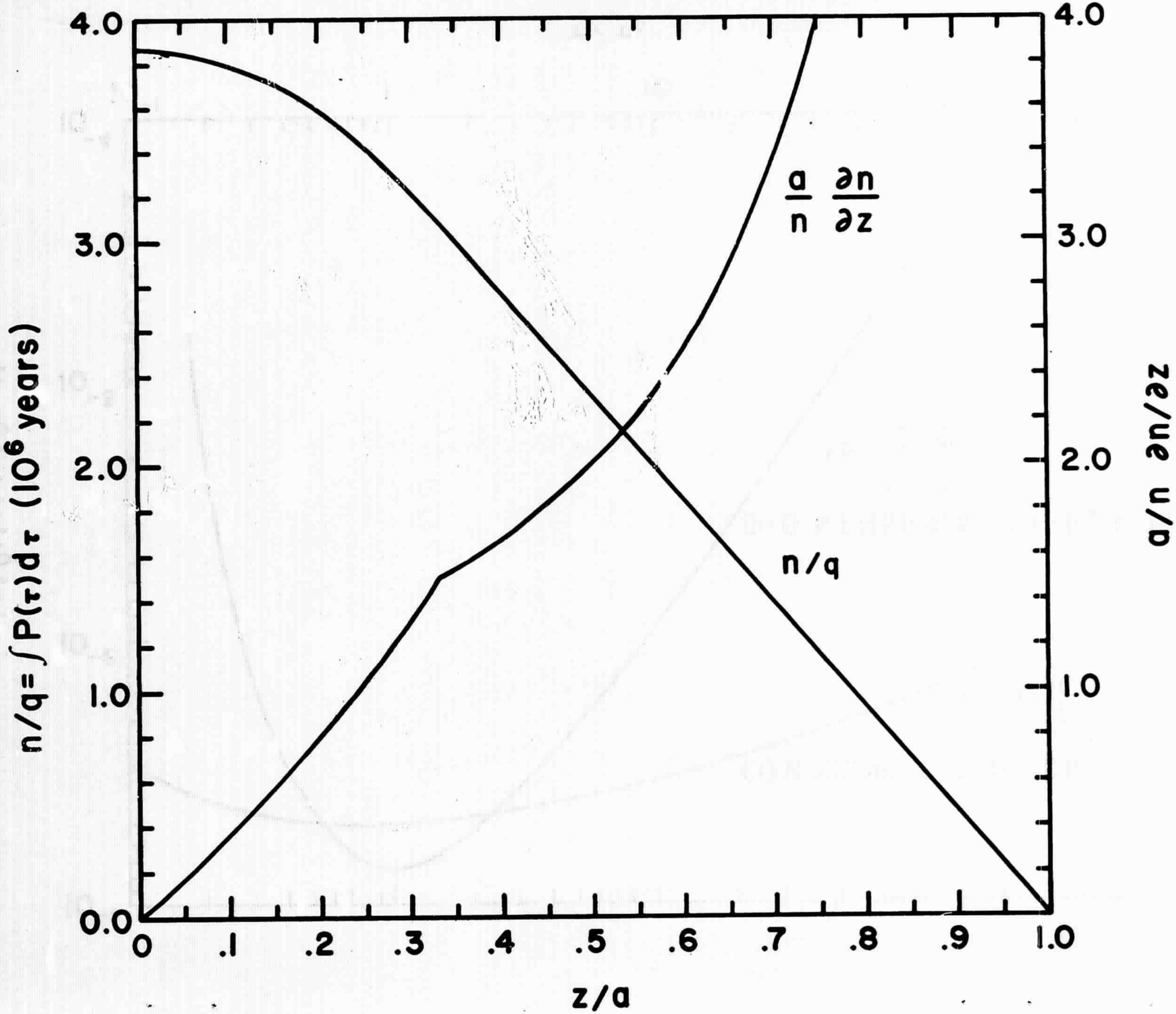
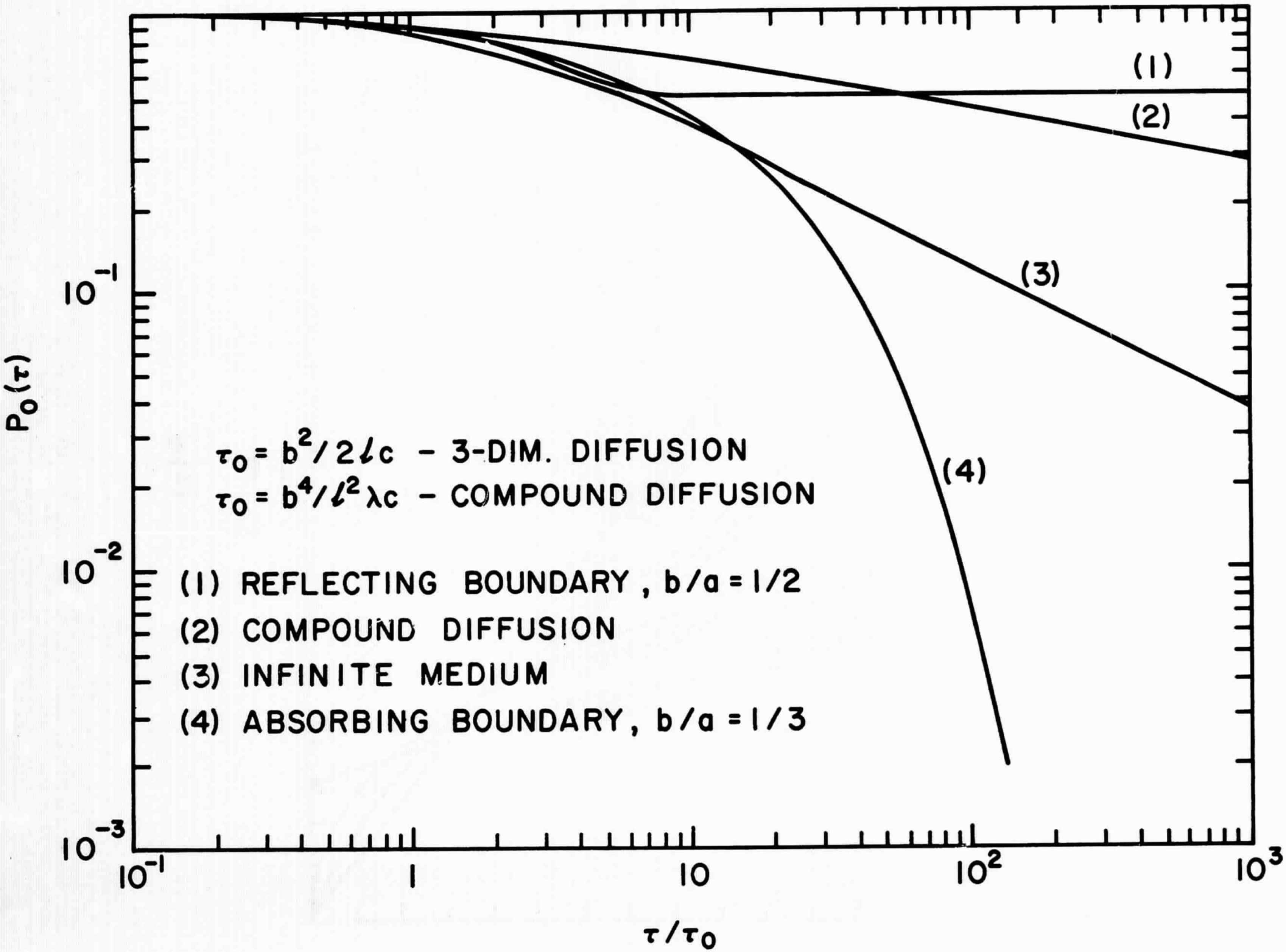


Figure 2



$P_0(\tau)$
τ/τ_0
10^{-1}
10^{-2}
10^{-3}
1
10
10^2
10^3
(1)
(2)
(3)
(4)
$\tau_0 = b^2/2\ell c$ - 3-DIM. DIFFUSION
$\tau_0 = b^4/\ell^2\lambda c$ - COMPOUND DIFFUSION
(1) REFLECTING BOUNDARY, b/a = 1/2
(2) COMPOUND DIFFUSION
(3) INFINITE MEDIUM
(4) ABSORBING BOUNDARY, b/a = 1/3

Figure 3

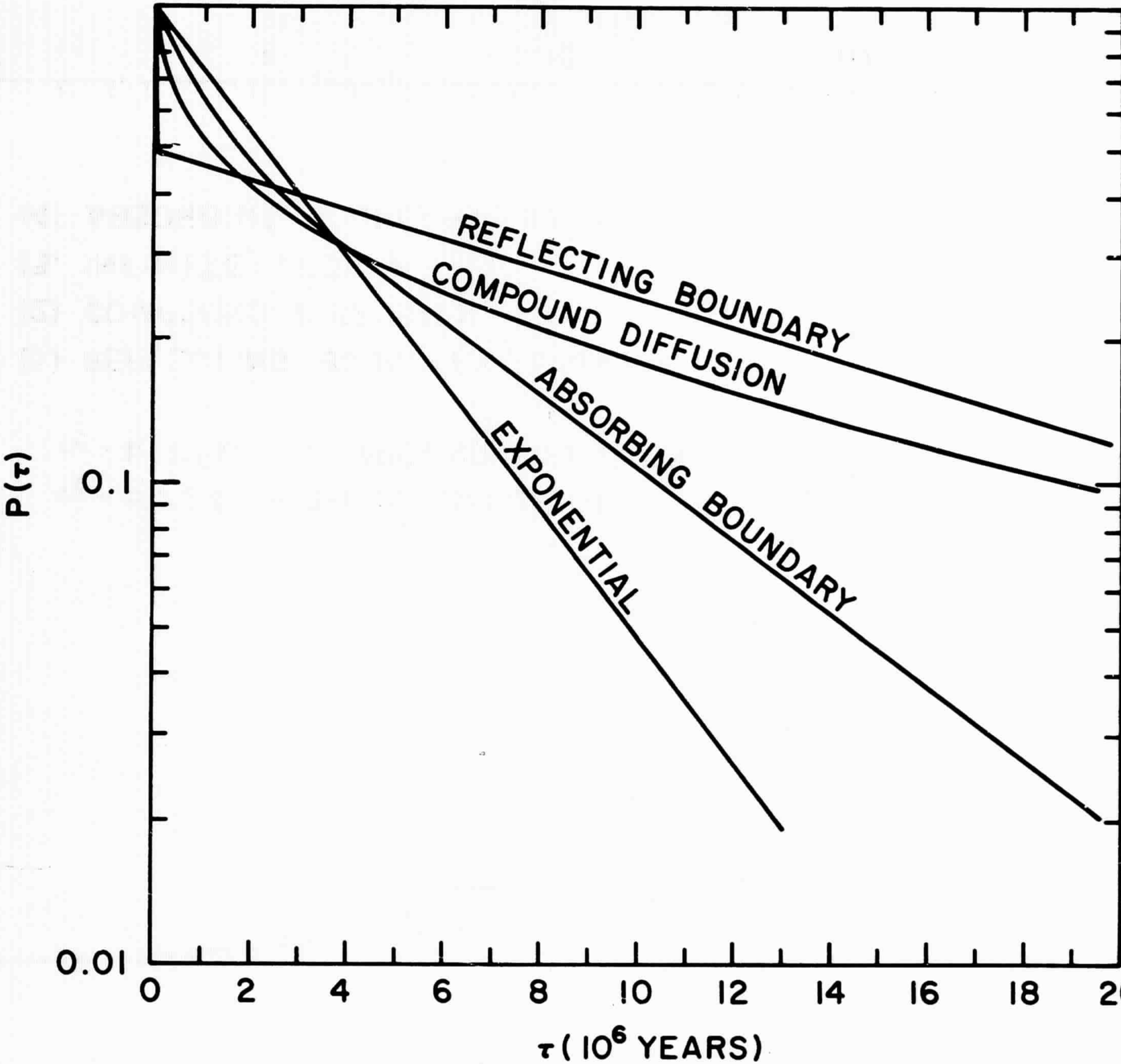

P(τ)
0.1
0.01
0
2
4
6
8
10
12
14
16
18
20
τ (10^6 YEARS)
REFLECTING BOUNDARY
COMPOUND DIFFUSION
ABSORBING BOUNDARY
EXPONENTIAL

Figure 4

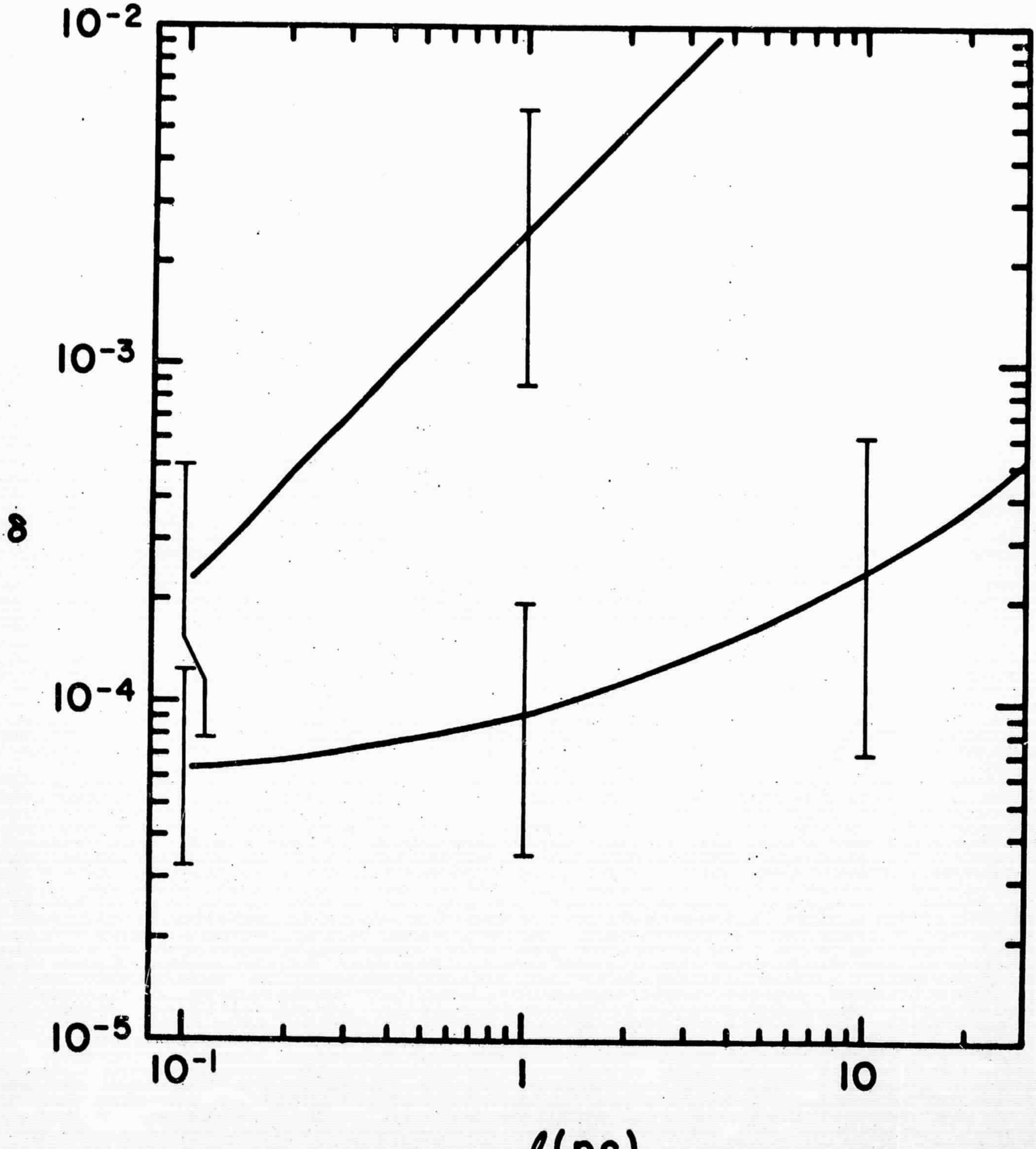


Figure 5